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1996 J. Phys.: Condens. Matter 8 10219

(http://iopscience.iop.org/0953-8984/8/49/013)

J. Phys.: Condens. Matter **8** (1996) 10219–10229. Printed in the UK

Non-local dynamics of weakly nonlinear spin excitations in thin ferromagnetic films

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Received 12 March 1996, in final form 23 July 1996

Abstract. Effective integro-differential equations of weakly nonlinear dynamics describing the interaction of quasi-one-dimensional exchange-dipole spin-waves are derived for a thin ferromagnetic slab (film). The non-local part of the magnetostatic dispersion of these waves has been taken into account. *Algebraic* soliton-like states have been predicted. The conditions of their existence and their dynamic properties are investigated depending on the film thickness and on the magnitude and orientation of the external magnetic field. The role of crystallographic magnetic anisotropy in the formation of these states is analysed.

1. Introduction

In recent years considerable interest has been devoted to the study of features of excitation and propagation of nonlinear spin waves in magnetic films. These films are convenient model systems for the study of linear spin-wave processes and modelling of nonlinear phenomena in dispersive media. One of the main results of this study is the detection of the envelope solitons under the conditions of pulse excitation and the propagation of dipole and exchange-dipole spin waves [1–4]. As a rule, a local nonlinear Schrödinger equation (NSE) is used for the adequate description of weakly nonlinear dynamics [5, 6]. A considerable body of work has been devoted to the analysis of its solutions under different initial and boundary conditions, among which are works on numerical computation [7, 8]. Usually this equation is deduced with the assumption that the system studied has a dispersion law depending on the amplitude of a spin wave. The approach requires the differentiability of this law over the entire range of variation of the wavenumber k. However, in the range of small wavenumbers $k(|kd| \to 0, d$ is the thickness of the film) the dispersion law is a non-differentiable function and, consequently, the approach used is not correct. Therefore, the applicability of a local NSE is limited to the range of moderate and large values of wavenumbers k.

The main aim of our paper is to derive the effective equations describing weakly nonlinear interaction of quasi-one-dimensional spin waves in a thin ferromagnetic slab in the long-wave limit, when the characteristic size of the magnetic inhomogeneities is very large compared with the slab thickness. In this interval of the wavenumbers the non-local character of the relation between the magnetization and magnetic field due to dipole–dipole interaction cannot be neglected. Effective equations of spin-wave dynamics have proved to be integro-differential. They are not reducible to the local NSE. Taking into account the non-local part of the magnetostatic spin-wave dispersion leads to the conclusion that, in thin ferromagnetic films, there are no purely *exponential* solitons in the long-wave

0953-8984/96/4910219+11$19.50

approximation. It has been shown at the same time that, under specific conditions, the exchange-magnetostatic dispersion and the spin-wave interaction permit the formation of localized states of the *algebraic* soliton type.

To derive the effective nonlinear equations, we use an approximation implying that the local dipole fields are replaced by the averaged ones over the film thickness. This approach is suitable for derivation of the effective equations of exchange-dipole mode dynamics. It can be applied to the case of thin films with free spins at the surface, if the distribution of the magnetization along the normal to the surface of the film is close to being a uniform one. Our approximation allows us to extract the branch of the spin waves with the lowest energy propagating along the slab.

The present paper consists of six sections. In section 2 the approximate method, taking into account the magnetostatic field, is described. With this method the dispersion law of linear spin waves is derived. It is well known that the dispersion law of linear spin waves is determined by the ground state of a spin system. The specific features of the weakly nonlinear spin excitations are determined by analytical properties of this dispersion law. Because of this, we should concentrate our attention on the dispersion law in this section.

Using these results, in section 3 the simplified equations of non-local dynamics of small-amplitude spin excitations are deduced in the case of a ferromagnetic slab. Section 4 is devoted to the analysis of possible soliton states. In section 5 the role of uni-axial magnetic anisotropy in the formation of *algebraic* solitons is discussed.

2. Effective de-magnetizing fields and the dispersion law of linear spin waves

Consider an isotropic ferromagnetic film (slab) of thickness d along the z axis magnetized by a uniform magnetic field $\boldsymbol{H}_0$ directed along the normal to the surface (namely along the z axis) or tangentially (along the y axis). The slab is under magnetic saturation conditions. The equation of motion of the magnetization $\boldsymbol{M}$ has the form

$$\partial_t \boldsymbol{M} = -|\gamma|[\boldsymbol{M} \times (\boldsymbol{H}_0 + \boldsymbol{H}^{(m)} + \alpha \Delta \boldsymbol{M})] \tag{1}$$

where $\partial_t \equiv \partial/\partial t$; $\boldsymbol{M}^2 = M_0^2$, M_0 is the saturation magnetization, γ is the magneto-mechanical ratio, α is the exchange interaction constant and Δ is the Laplacian operator. Below, the modulus sign around γ is omitted. The analysis is performed for the case of free spins at the surface of the slab, that is

$$\partial_z \boldsymbol{M}|_{z=\pm d/2} = 0\,. \tag{2}$$

The de-magnetizing field $\boldsymbol{H}^{(m)}$ satisfies the equations of magnetostatics

$$\operatorname{rot} \boldsymbol{H}^{(m)} = 0 \qquad \operatorname{div}(\boldsymbol{H}^{(m)} + 4\pi \boldsymbol{M}) = 0 \tag{3}$$

and continuity conditions of tangential components of the magnetic field vector $\boldsymbol{H}^{(m)}$ and the normal component of the induction vector $\boldsymbol{B}^{(m)} = \boldsymbol{H}^{(m)} + 4\pi \boldsymbol{M}$ at the boundary of a ferromagnet:

$$\left(\boldsymbol{H}_+^{(m)}\right)_\tau = \left(\boldsymbol{H}_-^{(m)}\right)_\tau \qquad \left(\boldsymbol{H}_+^{(m)}\right)_\nu + 4\pi M_\nu = \left(\boldsymbol{H}_-^{(m)}\right)_\nu \tag{4}$$

where the indices $(+)$ and $(-)$ denote the fields inside and outside the ferromagnetic slab, respectively, and the indices τ and ν are the tangential and normal components of the vectors $\boldsymbol{H}^{(m)}$ and $\boldsymbol{M}$ at the surface of a slab. The solution of the magnetostatic equation (3) with the boundary conditions (4) has the form [9]

$$\boldsymbol{H}^{(m)} = -\nabla\varphi \qquad \varphi = \int_v \mathrm{d}\boldsymbol{r}'\, M_i(r')\,\frac{\partial}{\partial x_i'}\,\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} \tag{5}$$

where the integration is performed over the slab's volume.

Consider the application of the relation (5) to the one-dimensional problem. To describe the quasi-one-dimensional motion of magnetization waves along the slab it is convenient to introduce the coordinate system $\xi 0 \eta$ in the plane of the slab instead of the coordinate system $x0y$:

$$\xi = x\cos\zeta + y\sin\zeta \qquad \eta = -x\sin\zeta + y\cos\zeta\ . \tag{6}$$

In this coordinate system the 0ξ axis coincides with the direction of wave propagation. Here ζ is the angle of the $0x$ axis with respect to the 0ζ axis. It is measured counter-clockwise from the $0x$ axis. Then the magnetization vector depends only on the two spatial coordinates: $\boldsymbol{M} = \boldsymbol{M}(\xi, z)$. The variable ξ defines the change in the magnetization in the slab plane. This allows one to perform the convenient Fourier transformation of the magnetization

$$\boldsymbol{M}(\xi, z) = \int \mathrm{d}k\, \boldsymbol{M}(k, z)\exp(\mathrm{i}k\xi)\ . \tag{7}$$

As mentioned above, we are interested in the exchange-dipole modes of the lowest type for which de-magnetizing fields and explicit formulae for the spin-wave spectrum can be approximated by a simple method. It is known that, in a slab with free spins at the surface the distribution of the magnetization along the normal to the surface of the slab is close to a uniform one [10] provided that the characteristic size of the magnetic inhomogeneity is much larger than the slab thickness d $(\lambda \gg d)$. By taking into consideration this fact, neglecting the dependence of the magnetization on the z coordinate and averaging the magnetostatic fields over the slab thickness

$$\langle \boldsymbol{H}^{(m)} \rangle = d^{-1}\int_{-d/2}^{+d/2} \boldsymbol{H}^{(m)}\, \mathrm{d}z \tag{8}$$

we find finally

$$\begin{pmatrix} \langle H_x^{(m)} \rangle \\ \langle H_y^{(m)} \rangle \end{pmatrix} = -4\pi \begin{pmatrix} \cos\zeta \\ \sin\zeta \end{pmatrix} \hat{S} M_\xi \qquad \langle H_z^{(m)} \rangle = -4\pi M_z(\xi) + 4\pi \widehat{S} M_z \tag{9a}$$

where

$$\widehat{S}u = \int \mathrm{d}\xi'\, s(\xi - \xi')u(\xi') \qquad s(\xi) = (2\pi)^{-1}\int \mathrm{d}k\, \exp(\mathrm{i}k\xi)s(k)$$
$$s(k) = 1 + |kd|^{-1}[\exp(-|kd|) - 1] \qquad M_\xi = M_x\cos\zeta + M_y\sin\zeta\ . \tag{9b}$$

An analogous result is obtained if the dipole energy is averaged,

$$E^{(m)} = -(2d)^{-1}\int_{-d/2}^{+d/2} \boldsymbol{H}^{(m)}\boldsymbol{M}\, \mathrm{d}\xi\, \mathrm{d}z$$

and the effective magnetostatic fields are defined by the relation $\langle \boldsymbol{H}^{(m)} \rangle = -\delta E^{(m)}/\delta \boldsymbol{M}(\xi)$ [11]. Let us emphasize that in formula (9) the operator $\widehat{S}$ acts only on the non-uniform part of the magnetization vector, since $s(k)\delta(k) = 0$; hence $\widehat{S} \times \text{constant} = 0$.

In a thin slab in the long-wave approximation at $|kd| \ll 1$ the operator $\widehat{S}$ takes the form

$$\widehat{S}u \approx -(d/2)\partial_\xi \widehat{H}u + (d^2/6)\partial_\xi^2 \tag{10}$$

where $\widehat{H}$ is the Hilbert operator

$$\widehat{H}u = (\pi)^{-1}\mathrm{P}\int_{-\infty}^{+\infty} \frac{\mathrm{d}\xi'\, u(\xi')}{\xi' - \xi}\ .$$

The symbol P denotes the principal-value integral.

Consider the spin excitation spectrum. Let the slab be under magnetic saturation conditions. Thereby the uniform magnetic field is directed along the normal to the surface of the slab, that is, along the z axis. It is further assumed that $H_0 \gg 4\pi M_0$, since at $H_0 = 4\pi M_0$ the state of uniform magnetization is unstable (the sample consists of domains). Substituting (9) into (1) and linearizing the system of equations obtained with respect to small deviations of the magnetization, we obtain the following expression for the linear mode spectrum:

$$\omega^2 = (\omega_H + \gamma\alpha M_0 k^2)[\omega_H + \gamma\alpha M_0 k^2 + s(k)\omega_M] \tag{11}$$

where

$$\omega_H = \omega_H^0 - \omega_M \qquad \omega_H^0 = \gamma H_0 \qquad \omega_M = \gamma 4\pi M_0 .$$

Let us note that, in this geometry, the spectrum does not depend on the direction of wave motion in the slab plane. In the limit $|kd| \ll 1$ the dispersion law takes the form

$$\omega^2 \cong \omega_H^2 + \omega_H\omega_M \frac{|kd|}{2} + \omega_H k^2 \left(2\gamma\alpha M_0 - \frac{\omega_M d^2}{6}\right). \tag{12}$$

If the field is directed along the y axis then simple calculations result in the following expression for the spectrum:

$$\omega^2 \cong [\omega_H^0 + \omega_M - \omega_M s(k) + \gamma\alpha M_0 k^2][\omega_H^0 + \omega_M s(k)\cos^2\zeta + \gamma\alpha M_0 k^2] . \tag{13}$$

In the long-wave limit ($|kd| \ll 1$) relation (13) takes the form

$$\omega^2 \cong \omega_H^0(\omega_H^0 + \omega_M) + \frac{|kd|}{2}\omega_M(\omega_M\cos^2\zeta - \omega_H^0\sin^2\zeta) + k^2\big(\gamma\alpha M_0(2\omega_H^0 + \omega_M) - \tfrac{5}{12}(\omega_M d\cos\zeta)^2 + \tfrac{1}{6}\omega_M\omega_H^0(d\sin\zeta)^2\big) . \tag{14}$$

The expressions (11)–(14) coincide with the formulae for the spin-wave spectrum of the lowest type in a thin slab with free spins at the surface obtained in [12–14]. This suggests that the approximate method used in the calculation of the dispersion law is applicable to the linear processes. We propose to use this method for research into the nonlinear phenomena. According to [13, 14] this branch of the spectrum corresponds to the value $n = 0$. The condition $n = 0$ implies the absence of nodes of eigenfunctions describing the non-uniformity of the spin-wave amplitude along the normal to the surface of the slab. In thin slabs this branch of the spectrum is consistent with the lowest energy in the range of long wavelengths. It should also be noted that the dispersion relations (12) and (14) depend on $|\boldsymbol{k}| \equiv k$. For the film magnetized along the normal to the surface this fact is specially noted in [12]. As a result the functions $\omega(k)$ (see the relations (12) and (14)) become non-differentiable at $|\boldsymbol{k}| \to 0$. The presence of the modulus of the wavevector $\boldsymbol{k}$ in the dispersion law rather than its projection on the direction of wave motion changes substantially the effective equation of weakly nonlinear dynamics of these waves, which becomes non-local and not reducible to the NSE. Non-locality of the dynamics equation follows from (1) and (9). This result is a consequence of the long-range character of magnetostatic interactions.

The other critical point in the neighbourhood of which nonlinear excitations are not described by the local NSE is the point of inflection. Its occurrence is due to the competition of two types of spatial dispersion, namely exchange and magnetostatic. It is located in the range of moderate and large values of wavenumber k. Our preliminary analysis shows that the indicated peculiarity occurs for rather thin films. We plan to publish a detailed analysis of this situation in a forthcoming article.

The third peculiar region of the dispersion law is the region of the anomalous dispersion. It originates from the interaction of the spin modes with different numbers n. The conditions of the experimental manifestation of this peculiarity were analysed in [2]. At present there is no adequate theoretical model describing weakly nonlinear phenomena in this interval of wavenumbers k.

3. Interaction of spin excitations of small amplitude and evolution equations

Let us deduce the effective evolution equations of weakly nonlinear exchange-dipole waves of the lowest type in ferromagnetic films assuming that these waves propagate along the 0ζ axis and that $|kd| \ll 1$.

Consider first the quasi-one-dimensional dynamics of spin excitations in the slab magnetized along the normal to the surface. Write down the corresponding Landau–Lifshits equations for deviations of the magnetization from the ground state to an accuracy of cubic terms

$$\partial_t m_x = -[\omega_H - \alpha\gamma M_0 \partial_\xi^2 + \omega_M \sin^2(\zeta)\widehat{S}]m_y - \tfrac{1}{2}\omega_M \sin(2\zeta)\widehat{S}m_x - \omega_M (2M_0^2)^{-1} m_y(m_x^2 + m_y^2) \qquad (15a)$$

$$\partial_t m_y = [\omega_H - \alpha\gamma M_0 \partial_\xi^2 + \omega_M \cos^2(\zeta)\widehat{S}]m_x + \tfrac{1}{2}\omega_M \sin(2\zeta)\widehat{S}m_y + \omega_M (2M_0^2)^{-1} m_x(m_x^2 + m_y^2)\,. \qquad (15b)$$

In (15) in transforming the nonlinear terms we have taken into account that $m_z = -(2M_0)^{-1}(m_x^2 + m_y^2)$. Thereby the terms involving the derivatives with respect to ξ have been neglected, since they yield corrections of order $\mathrm{O}(d/\lambda)$ and $\mathrm{O}(d^2/\lambda^2)$ to the constant of wave interaction. Differentiating the equations of system (15) with respect to t and using the approximations $\partial_t m_x \approx -\omega_H m_y$ and $\partial_t m_y \approx \omega_H m_x$ in transforming the nonlinear terms we obtain the effective equation defining the field $\varphi = M_0^{-1}(m_x + \mathrm{i}m_y)$

$$\partial_t^2\varphi + \omega_0^2\varphi + a\partial_\xi \widehat{H}\varphi + b\partial_\xi^2\varphi + g|\varphi|^2\varphi = 0\,. \qquad (16)$$

Here

$$\omega_0^2 = \omega_H^2 \qquad a = -\omega_H\omega_M d/2 \qquad b = \omega_H[(\omega_M d^2/6) - 2\gamma\alpha M_0] \qquad g = \omega_H\omega_M\,. \qquad (17)$$

In transforming the dispersion terms in (18) we have used the approximation (10) for the operator $\widehat{S}$ and the relation $\widehat{H}^2 = -1$.

When the external magnetic field lies in the plane of the slab (to be specific, along the y axis), the spin-wave spectrum (14) depends on the direction of wave propagation along the slab. Moreover, the magnetostatic interactions give rise to the appearance of quadratic rather than cubic terms in amplitudes of magnetization deviations from the ground state in Landau–Lifshits equations

$$(\boldsymbol{M} = M_0\boldsymbol{n} + \boldsymbol{m}, \boldsymbol{n} = (0,1,0), m_y \approx -(2M_0)^{-1}(m_x^2 + m_z^2), H_0 \gg 4\pi M_0) :$$

$$\partial_t m_x = (\omega_M - \omega_M\hat{S} - \gamma\alpha M_0\partial_\xi^2 + \omega_H^0)m_z - 2\pi\gamma\sin(2\zeta)m_z\hat{S}m_x - \frac{\omega_M}{2M_0^2}m_z(m_x^2 + m_z^2)$$

$$\partial_t m_z = -[\omega_H^0 + \omega_M\cos^2(\zeta)\hat{S} - \gamma\alpha M_0\partial_\xi^2]m_x + 2\pi\gamma\sin(2\zeta)m_x\hat{S}m_x + \frac{\omega_M}{4M_0}\sin(2\zeta) \times\hat{S}(m_x^2 + m_z^2)\,.$$

The coefficients of the quadratic terms are proportional to $\sin(2\zeta)$ and, consequently, vanish for the waves propagating along the external magnetic field or perpendicularly to it. For the directions of wave and spatial scales λ satisfying the condition $(d/\lambda)\sin(2\zeta) \ll (m_i/M_0)$, $i = x, z$, the magnetostatic quadratic terms may be neglected. If the corrections of the order of $\mathrm{O}(d/\lambda)$ and $\mathrm{O}(\omega_M/\omega_H^0)$ to the constant of the wave interaction are also neglected, simple calculations analogous to those presented above result in an equation which coincides in form with (16), where, φ, ω_0^2, a, b and g should be now replaced by the following expressions, respectively:

$$\varphi = \frac{m_x + \mathrm{i}m_z}{M_0} \qquad \omega_0^2 = \omega_H^0(\omega_H^0 + \omega_M) \qquad a = \frac{d}{2}\omega_M(\omega_H^0 \sin^2\zeta - \omega_M \cos^2\zeta)$$
$$b = \frac{5(\omega_M d\cos\zeta)^2}{12} - \frac{\omega_M\omega_H^0(d\sin\zeta)^2}{6} - \gamma\alpha M_0(2\omega_H^0 + \omega_M) \qquad g = -\frac{\omega_M\omega_H^0}{2}\,. \tag{18}$$

The integro-differential equation (16) cannot be reduced to the local differential equation of the NSE type as in [5, 6]. Note, however, that, when the parameters characterizing the wave are changed sufficiently slowly over an interval of the order of the period of fast oscillations of the magnetization, equation (16) can be written in a form like the NSE. Let us illustrate this.

To separate the fast oscillation from the slow ones in (16) we can go from φ to a new variable Ψ (the function Ψ varies slowly with coordinate and time) according to the formula $\varphi = \Psi\exp(\mathrm{i}\omega_0 t)$. Then

$$\partial_t^2\varphi = (\partial_t^2\Psi + 2\mathrm{i}\omega_0\partial_t\Psi - \omega_0^2\Psi)\exp(\mathrm{i}\omega_0 t)\,. \tag{19}$$

It follows from (16) and (19) that, to a good approximation, we can write

$$\partial_t\Psi \approx -\frac{a}{2\mathrm{i}\omega_0}\partial_\xi\hat{H}\Psi$$

and, consequently, we have

$$\partial_t^2\Psi \approx \left(\frac{a}{2\mathrm{i}\omega_0}\right)^2\hat{H}^2\partial_\xi^2\Psi = \frac{a^2}{4\omega_0^2}\partial_\xi^2\Psi \tag{20}$$

since $\hat{H}^2 = -1$. Now express the first terms in (19) in terms of spatial derivatives of the field Ψ in accordance with (20). As a result of averaging over fast oscillations in time equation (16) takes the form

$$\mathrm{i}\partial_t\Psi + a_1\partial_\xi\hat{H}\Psi + b_1\partial_\xi^2\Psi + g_1|\Psi|^2\Psi = 0$$
$$a_1 = \frac{a}{2\omega_0} \qquad b_1 = \frac{1}{2\omega_0}\left[b + \left(\frac{a}{2\omega_0}\right)^2\right] \qquad g_1 = \frac{g}{2\omega_0}\,. \tag{21}$$

Equation (21) represents correctly the spin-wave spectrum (12). This equation is an analogue of the non-local NSE. To derive the NSE in [6], the authors have used the expansion of the nonlinear dispersion relation in powers of k and $|\Psi|^2$ near $k = |\Psi| = 0$ and the inverse Fourier transformation. Since the dispersion law depends on $|\boldsymbol{k}|$, such an expansion leads to a term of the form $-a_1|\boldsymbol{k}|\Psi(k)$ rather than $-a_1 k\Psi(k)$ as in [6]. Accordingly, the inverse Fourier transformation of this term is $a_1\partial_\xi\hat{H}\Psi$ rather than $\mathrm{i}a_1\partial_\xi\Psi$. In view of this remark the calculations of [6] lead to equation (21). Equation (16) is more general than (21) because it is not related to one rapidly oscillating harmonic and does not assume the slow time modulations of the spin-wave amplitude.

4. Soliton-like excitations in film

Equation (16) (or (21)) is not completely integrable, but it permits exact localized solutions like solitons. These solutions may be found by using the property of the Hilbert operator. Let $F_+(\xi)$ and $F_-(\xi)$ be meromorphic functions of the complex variable ξ. Thereby all the poles of these functions lie in the top ($\mathrm{Im}\,\xi > 0$) and bottom ($\mathrm{Im}\,\xi < 0$) half-planes, respectively, and $F_+(\xi) \to 0$, $F_-(\xi) \to 0$ at $|\xi| \to \infty$. Then using the Cauchy theorem about residues it is not difficult to show that the following relations are valid:

$$\hat{H}F_+ = -\mathrm{i}F_+ \qquad \hat{H}F_- = \mathrm{i}F_- . \tag{22}$$

The relations (22) allow us to seek the solution of equation (16) in the class of meromorphic functions. In particular, using in (16) the substitution

$$\varphi = \frac{A(t)}{\xi - \nu(t)} + \frac{B(t)}{\xi - \nu^*(t)} \qquad \mathrm{Im}\,\nu > 0 \tag{23}$$

and setting the coefficients of linear independent functions $(\xi - \nu^*)^{-m}$ and $(\xi - \nu)^{-m}$ ($m = 1, 2, 3$) equal to zero we obtain a system of ordinary differential equations for the complex functions A, B and ν. At some reductions this system can be proved to be compatible. Thus, at

$$ag > 0 \qquad bg > 0 \tag{24}$$

the following solution is possible:

$$\begin{gathered} A = -B = |C|\exp(\mathrm{i}\Omega t + \mathrm{i}\varphi_0) \qquad \mathrm{Re}\,\nu = 0 \qquad \mathrm{Im}\,\nu = \frac{3b}{a} \equiv \Delta \\ \Omega^2 = \omega_0^2 + \frac{a^2}{3b} \qquad |C|^2 = \frac{2b}{g} \\ \varphi = \frac{2\mathrm{i}\Delta|C|\exp(\mathrm{i}\Omega t + \mathrm{i}\varphi_0)}{\xi^2 + \Delta^2} . \end{gathered} \tag{25}$$

It corresponds to a precessing soliton. Here φ_0 is the real parameter.

The analysis shows that if the film is magnetized perpendicularly to its boundaries, no soliton regimes seem to be possible. The conditions of soliton existence (24) are more easily satisfied if the film is magnetized tangentially (see equation (18)). This is possible, if $\tan^2\zeta < \omega_M/\omega_H^0 \ll 1$. In other words, the geometry in which the spin waves propagate along the boundary of the slab, namely perpendicularly to the magnetic field, is favourable for the formation of solitons. Since in the tangentially magnetized film the condition $g < 0$ holds and the condition $bg > 0$ can be fulfilled only in the presence of sufficiently intensive exchange interactions, there are no purely magnetostatic soliton states in such a film. It is not difficult to show also that the condition $bg > 0$ imposes the following restriction on the slab thickness d:

$$d < a_0[H_E H_0/(2M_0^2)]^{1/2} \qquad H_E = \alpha M_0 a_0^{-2}$$

where a_0 is the lattice constant. If we adopt for yttrium ferrite $H_E \approx 10^4$ Oe, $M_0 \approx 140$ G and assign 2×10^3 Oe to H_0 [15], then we obtain $d < 22.5a_0$. For these values of parameters the soliton size Δ proves to be larger than the slab thickness ($\Delta \gg d$).

The Lagrange function with the density

$$L_{eff} = \vartheta[|\partial_t\varphi|^2 - \omega_0^2|\varphi|^2 - a\varphi^*\partial_\xi\hat{H}\varphi + b|\partial_\xi\varphi|^2 - (g/2)|\varphi|^4] \tag{26}$$

can be associated with equation (16). Here $\vartheta = M_0/(2\gamma\tilde{\omega})$, where $\tilde{\omega} = \omega_H$ and $\tilde{\omega} = \omega_H^0$ correspond to the orientation of the external magnetic field along the normal to the slab and along the y axis, respectively. The formula (26) allows us to find the soliton energy (27):

$$E = \vartheta \int_{-\infty}^{+\infty} \mathrm{d}\xi \, [|\partial_t \varphi|^2 + \omega_0^2 |\varphi|^2 + a\varphi^* \partial_\xi \hat{H}\varphi - b|\partial_\xi \varphi|^2 + (g/2)|\varphi|^4]$$
$$= \Omega I + E_0 \qquad E_0 = -\frac{5\pi M_0 a^3}{27\gamma\omega_H^0 g b} > 0 \,. \tag{27}$$

The soliton energy (27) is expressed in terms of the adiabatic invariant I of precessional motion:

$$I = \frac{\vartheta}{\pi} \int_{-\infty}^{+\infty} \mathrm{d}\xi \int_0^T \mathrm{d}t \, |\partial_t \varphi|^2 = \frac{4\pi M_0 \Omega a}{3\gamma\omega_H^0 g}$$

where $T = (2\pi/\Omega)$ is the precession period. Such a form allows one to perform the quasi-classic quantization of the soliton energy setting $I = \hbar N$, where N is an integer. It also allows one to treat the algebraic soliton (25) as a complex consisting of N non-interacting particles with energy $\hbar\Omega$. It is of interest to note that the frequency of soliton precession Ω which is defined by the spin-wave interaction proves to be smaller than the frequency of the uniform resonance of linear modes, that is $\Omega < \omega_0$.

Note that, in the case considered, the soliton size Δ, the frequency of its precession Ω, the energy E and the invariant I are unambiguously defined by the external conditions, namely the slab thickness, the magnitude of the magnetic field and so on. It is not difficult to show that the form of equation (16) and its solution (25) are not changed, if there is a strong enough uni-axial anisotropy in the plane of the slab (the anisotropy axis is perpendicular to the direction of propagation of the spin waves). The corresponding solution of the Landau–Lifshits equations has been obtained earlier in [16] with the help of asymptotic perturbation theory and numerical methods.

5. Algebraic soliton-like states in a ferromagnetic film with an ‘easy-plane’ anisotropy

To illustrate the role of crystallographic magnetic anisotropy, we consider the case of a ferromagnetic slab with ‘easy-plane’ anisotropy. Let the anisotropy axis be perpendicular to the normal to the plane of the slab and a sufficiently weak magnetic field H_0 be applied in this plane:

$$H_0 \ll M_0, \; (\beta + 4\pi)M_0 \qquad \beta > 0 \,. \tag{28}$$

Here β is the anisotropy constant. It is readily seen that the geometry in which the field H_0 is perpendicular to the direction of spin-wave propagation is favourable for the formation of soliton states. Assume that the waves propagate along the x axis and the magnetic field is directed along the y axis. Then the magnetic energy density can be written in the form

$$w = \tfrac{1}{2}\alpha(\partial_x M)^2 + \tfrac{1}{2}\beta M_z^2 - H_0 M_y - \tfrac{1}{2}\boldsymbol{M} \cdot \boldsymbol{H}^{(m)} \,. \tag{29}$$

In describing the magnetostatic waves with the scale of inhomogeneity being much greater than the slab thickness, it is convenient to use the expression (9). Let us introduce the following parametrization for the magnetization vector $\boldsymbol{M}$, namely $\boldsymbol{M} = M_0(\cos\theta\cos\psi, \cos\theta\sin\psi, \sin\theta)$. Then the dynamics equation can be obtained by variation of the action corresponding to the Lagrange function with the density with respect to the field θ, ψ:

$$L = \frac{M_0}{\gamma} \sin\theta \, \partial_t \psi - w \,.$$

Since we are interested in the small-amplitude waves, we write down the dynamic equation to an accuracy of the terms cubic in the deviation $\tilde{\psi}, \tilde{\theta}$ from the ground state ($\psi = \pi/2, \theta = 0$). In the long-wave limit the components with spatial derivatives in these terms may be neglected. In the presence of crystallographic 'easy-plane' anisotropy the angle $\tilde{\theta}$ characterizing the departure of the magnetization vector from the film plane is small in comparison with the angle $\tilde{\psi}$ which defines the oscillations of M in the x–y plane:

$$\frac{\tilde{\theta}}{\tilde{\psi}} \approx \left(\frac{H_0}{M_0(\beta + 4\pi)}\right)^{1/2} \ll 1\,. \tag{30}$$

According to the condition (30) the nonlinear terms involving $\tilde{\theta}$ may be ignored. As a result the system of dynamic equations takes the simple form:

$$\begin{aligned} &\frac{\partial_t \tilde{\psi}}{\gamma M_0} - (\beta + 4\pi + h_0 - 4\pi\hat{S} - \alpha\partial_x^2)\tilde{\theta} = 0 \\ &\frac{\partial\tilde{\theta}_t}{\gamma M_0} + (h_0 + 4\pi\tilde{S} - \alpha\partial_x^2)\tilde{\psi} - \frac{h_0}{6}\tilde{\psi}^3 = 0 \qquad h_0 = H_0/M_0\,. \end{aligned} \tag{31}$$

Eliminating $\tilde{\theta}$ from the system of equations (31) we obtain the closed effective equation in terms of $\tilde{\psi}$:

$$\begin{aligned} &\partial_t^2\tilde{\psi} + \omega_0^2\tilde{\psi} - \rho\partial_x\hat{H}\tilde{\psi} - \mu\partial_x^2\tilde{\psi} - q\tilde{\psi}^3 = 0 \\ &\omega_0^2 = (\gamma M_0)^2 h_0(\beta + 4\pi + h_0) \qquad \rho = (\gamma M_0)^2 2\pi d(\beta + 4\pi) \\ &q = \frac{(\gamma M_0)^2}{6} h_0(\beta + 4\pi + h_0) \qquad \mu = (\gamma M_0)^2\left[\alpha(\beta + 4\pi + 2h_0) - \tfrac{1}{3}d^2 2\pi(\beta + 10\pi)\right]. \end{aligned} \tag{32}$$

In calculating the dispersion terms in (32) we have used the approximation (10). It is interesting that the interval of parameter variation in (32) permits a solution like (23). Thereby the difference of solution (23) from (25) is that the soliton obtained is moving:

$$\begin{aligned} &\tilde{\psi} = \frac{2c\Delta_0}{(x - vt)^2 + \Delta_0^2} \qquad c^2 = \frac{2\rho^2}{3q\omega_0^2} \qquad \Delta_0 = \frac{\rho}{\omega_0^2} \approx \frac{2\pi d}{h_0} \gg d \\ &v^2 = -\frac{1}{3}\left(\frac{\rho}{\omega_0}\right)^2 + \mu \approx (\gamma M_0)^2 a_0^2(\beta + 4\pi)\left[\frac{H_E}{M_0} - \frac{1}{3}\left(\frac{2\pi d}{a_0}\right)^2\frac{M_0}{H_0}\right] > 0\,. \end{aligned} \tag{33}$$

The last inequality in (33) assumes the obligatory presence of exchange interactions and imposes a restriction on the slab thickness and the magnitude of the external magnetic field, namely that the field should not be too small. It is seen that the reduction in H_0 leads to a drop in the soliton's velocity and to an increase in its width. At

$$d \to d_{crit} = a_0\left(\frac{3H_E H_0}{(2\pi)^2 M_0^2}\right)^{1/2}$$

the velocity of soliton motion approaches zero. Let us estimate the velocity (33). Setting the slab thickness $d = d_{crit}/3$ and taking into consideration the following values of parameters typical for the yttrium ferrite $H_E \approx 10^4$ Oe, $M_0 \approx 140$ G, $\beta \approx 30$, $a_0 \approx 12$ Å, $\gamma \approx 1.76 \times 10^7$ rad s^{-1} Oe^{-1} we obtain $v \approx 130$ m s^{-1} [15, 17]. This value is high since the velocity depends by definition on the exchange field and the magnitude of the lattice constant a_0 is sufficiently large in the case of yttrium ferrite.

The effective equation (32) can be obtained by variation of the action corresponding to the Lagrange function with the density

$$L_{eff} = \vartheta_1\left((\partial_t\tilde{\psi})^2 - \omega_0^2\tilde{\psi}^2 + \rho\tilde{\psi}\partial_x\hat{H}\tilde{\psi} - \mu(\partial_x\tilde{\psi})^2 + \tfrac{1}{2}q\tilde{\psi}^4\right). \tag{34}$$

Here $\vartheta_1 = [2\gamma^2(\beta + 4\pi + h_0)]^{-1}$. The energy and field momentum of a system have the following forms, respectively:

$$\begin{aligned} E &= \vartheta_1 \int_{-\infty}^{+\infty} \mathrm{d}x \big((\partial_t \tilde{\psi})^2 + \omega_0^2 \tilde{\psi} - \rho \tilde{\psi} \partial_x H \tilde{\psi} + \mu (\partial_x \tilde{\psi})^2 - \tfrac{1}{2} q \tilde{\psi}^4\big) \\ &= Pv + \frac{10\pi \omega_0^2 \rho \vartheta_1}{9q} \end{aligned} \tag{35}$$

$$P = 2\vartheta_1 \int_{-\infty}^{+\infty} \mathrm{d}x\, \partial_t \tilde{\psi} \partial_x \tilde{\psi} = \frac{4\pi \vartheta_1 \omega_0^4 v}{3\rho q}. \tag{36}$$

As in the case of solution (25), external conditions define the integrals of motion of the localized state uniquely. Note that the *algebraic* solitons analogous to (33) also occur in antiferromagnetic films with an 'easy-plane' anisotropy [11, 18]. However, in the latter case they are described by the integrable Benjamin–Ono model.

6. Conclusions

The analysis of dynamics of weakly nonlinear exchange-dipole spin excitations in thin ferromagnetic films (slabs) shows that, in the long-wave approximation ($|kd| \ll 1$), the effective equations of evolution are integro-differential and are not reduced to the nonlinear local Schrödinger equation. Moreover, these equations are not completely integrable. It has been shown that in ferromagnetic slabs in the long-wave region there are no purely magnetostatic *exponential* solitons. At the same time, the system considered permits the existence of exchange-dipole states of the *algebraic* soliton type. The non-local part of the magnetostatic dispersion 'smooths' out the spatial inhomogeneities of the magnetization distribution. The role of the external magnetic field is thereby proved to be extremely important. Namely, the considered localized states occur only in tangentially magnetized slabs, provided that the direction of the wave propagation is perpendicular to the direction of magnetic field. What is more, the formation of the precessing algebraic soliton has made possible the energy resonance absorption at a frequency smaller than the frequency of the uniform resonance of linear modes (uniform ferromagnetic resonance). The experimental detection of this phenomenon might be conclusive evidence for the occurrence of this soliton-like state. A good argument in support of this effect might also be the experimental observation of the field dependence of the frequency of the new resonance. It should be noted that the crystallographic magnetic anisotropy is also responsible for nonlinearity. For example, if in the case of a ferromagnet with 'easy-plane' anisotropy such an anisotropy is taken into account, the *algebraic* soliton acquires non-zero velocity. However, in these systems the conditions for the existence of solitons are more stringent. In particular, essential restrictions on the slab thickness are imposed.

Acknowledgment

This work was supported in part by the Russian Science Foundation under grant 93-02-2011.

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